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STABILITY OF SOLVING OPTIMIZATION PROBLEMS OF PARAMETERS OF RADIATIVE HEATING DEVICES

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Results are given of a study of correctness of problems related to the determination of optimal parameters of radiative heating devices from a given flux field of incident radiation.

Computational-theoretical analysis of construction arrangement schemes of radiative heating devices (RHD) is one of the problems solved during the preparation of thermal tests of materials and structures on benches of a radiative heater [1]. Similar studies are required for the determination of such (optimal) RHD parameters (the spatial location of the emitter, the screen shape, etc.), for which the realized conditions of thermal loading on the surface of the tested product corresonds fully to the given one.

From the point of view of the theory of inverse problems (IP) of heat transfer the search for optimal RHD parameters belongs to the class of inverse problems of radiative heat transfer (IPRHT) of the projection type. Similar problems are naturally formulated as extremal, and to solve them one uses methods of the theory of mathematical programming [2]. A specific feature of inverse problems consists of the fact that in the general case they may be incorrect, as a consequence of which the condition of uniqueness and (or) stability of the solution with respect to small varying input data may be violated [3].

In [4] was realized a parametric optimization of RHD, consisting of three sources of radiation and a planar screen, by means of one of the direct methods of nonlinear programming, the method of sliding tolerance. In this case of special studies no verification was carried out of the correctness of the statement of the problem mentioned. The purpose of the present study is a more detailed analysis of the given problem.

For this we consider the geometric IPRHT, the unknowns in which being the vertical coordinates of radiative line sources, for which the distribution $E_{inc}(x)$ on a plate of infinite length and finite width is closest to a uniform distribution with a given density E_{inc}^0 = const. The distances between all radiators over horizontal directions were taken identical, while the peripheral radiators, independently of their total number, were located over the edges of the plate. The extent of nonuniformity of $E_{inc}(x)$ was characterized by the quantity

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TABLE 1. Original Data and Solution Results of Geometrical IPRHT

No. of variation	$n\ (N+1)$	$y_i^0\ (i=1,\ n)$	t_0	E_{inc}^0	$\Phi_{min}\cdot 10^3$	δE, %	m_f
1	11	0,5	0,1	0,2	6,04	0,41	624
2	11	0,5	0,01	0,2	4,38	0,34	283
3	11	1,0	0,1	0,21	4,03	0,31	271
4	11	0,5	0,1	0,2	4,71	0,34	532
5	6	0,5	0,1	0,2	6,87	0,36	57
6	11	0,5	0,1	0,2	3,77	0,19	470
7	16	0,5	0,1	0,2	2,64	0,13	180
8	6	0,5	0,1	0,2	6,46	0,34	39
9	11	0,5	0,1	0,2	2,99	0,16	100
10	16	0,5	0,1	0,2	1,75	0,08	133
11	6	0,5	0,1	0,2	6,46	0,34	35
12	11	0,5	0,1	0,2	2,98	0,15	36
13	16	0,5	0,1	0,2	1,77	0,08	41

$$\delta E = \max |E_{inc}(x) - E_{inc}^0|/E_{inc}^0 \cdot 100\,\%.$$

As in [4], as purpose functional (PF) we took the mean square deviation of the calculated field $E_{inc}(x)$ from the given E_{inc}^0: $\Phi = \left(\int_l (E_{inc}(x)/E_{inc}^0 - 1)^2\, dx\right)^{1/2}$.

Since in the problem considered $E_{inc}(x)$ depends only on the vertical coordinates of radiators, the purpose functional provides some (implicit) dependence on the varying parameters. Thus, Φ satisfies the general principles of the statement of extremal functions [5]: Its value is determined only by the input data, not varying during the solution, and by the varying parameters; the minimum of Φ is reached for values of the varying parameters corresponding to the unknown solution. As in [4], to minimize Φ we used the method of sliding tolerance. The determination of $E_{inc}(x)$ was carried out by the method discussed in [6].

The simplicity of numerical realization of the method used and, as a consequence, the short computer solution time of the direct problem of radiative heat transfer made it possible to carry out detailed studies of the effect on obtaining stable results of various IP statements, as well as several discretization parameters of the mathematical model and of the optimization method.

With the purpose of analyzing the correctness of the original IPRHT statement we carried out a series of calculations, in which we varied the following input data: the number of radiators n = 6, 11, and 16 (we respectively also varied the number of zones by which the heated surface was divided N = n − 1); the initial size of the multiple boundary t_0 (one of the parameters in the sliding tolerance method); the initial search point y_i^0, $i = \overline{1, n}$; and the assigned flux density of the incident radiation E_{inc}^0. The finite deformation size of the multiple boundary was taken to be 10^{-4} in all cases considered. The numerical values found without account of a priori information for the minimum of the purpose functional Φ_{min}, of the nonuniformity δE and of the final number of steps of optimization search m_f (the number of solutions of the direct problem) for n = 11 are shown in Table 1 (the first four variations). The corresponding ordinates y_i of the radiators are shown in Fig. 1 (the digits at the curves of the figures correspond to the numbers of computational variations in Table 1). For smoothness the optimal coordinates of radiator locations are combined by segments of straight lines. All results are shown in dimensionless form, for which the geomet-

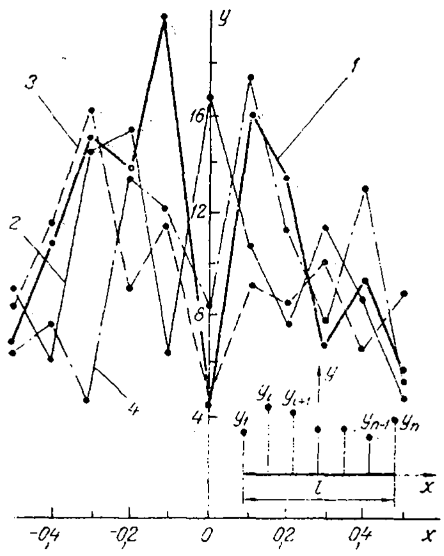

Fig. 1

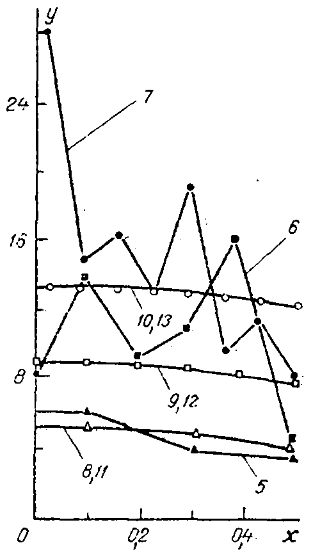

Fig. 2

Fig. 1. Computational scheme and optimal coordinates of radiators, obtained by solving the inverse problem of radiative heat transfer without account of a priori information (n = 11).

Fig. 2. Optimal coordinates of radiators, found by solving the regularized statement of the problem.

ric sizes are with respect to the plate width, and the energy characteristics — to the power of a single radiator.

Even a qualitative estimate of the results obtained indicates strong dependence of the unknown solution on the input data.

Variation of method parameters (t_0) and of the initial search point (y_i^0) affected not only the solution time, but also its final result, implying violation of the uniqueness condition. Small perturbations of the input data (E_{inc}^0) led to substantial changes in the solution, making it possible to draw conclusions concerning nonsatisfaction of the stability condition. It is also necessary to note that all "curves" obtained, on which radiators are located, are nonsmooth and nonsymmetric with respect to the middle of the plate; this is in bad agreement with the intuitive concept of optimal distribution of radiator heights (on the main symmetric curve, somewhat lowered toward the edges of the plate).

Similar results were also obtained for a number of radiators n = 6 and 16, which qualitatively coincide completely with the case considered n = 11.

An important aspect is the good observed optimization convergence. In all cases the optimum search was carried out by the condition of achieving a given accuracy, and this implies that the deformation of multiple boundaries at the last steps of the search were found near the minimum of the PF. The dependence of the solution on the initial search point can be explained by the coincidence of the multiple boundary in the region of various local minima, which implies multiple extremum nature of the PF surface. We also note that the different values of Φ_{min} at points of various local minima are quite small.

To obtain the regularized IPRHT solutions an approach is used, based on account of the a priori information, and consisting of searching solutions on a manifold narrower than the original range of allowed values of independent variables. In this case, depending on the method of assigning this set, the original incorrect problem can be transformed into a conditionally correct one [3]. Three series of problems were solved by using this approach: In the second series (variations 5-7) the symmetric location of radiators was assigned a priori, and in the third (variations 8-10) — additional restrictions of the form $y_{i+1} \geq y_i$ ($i = 1, 2, \ldots, n/2$) for n, $i = 1, 2, \ldots, (n+1)/2$ for odd n were imposed on the radiator ordinates. A condition of radiator location on the parabola $y = x^2 + c$, whose coefficients were also determined by minimizing the PF, was assigned in the fourth series of calculations (variations 11-13).

As seen from Fig. 2, imposing the symmetry condition leads to some decrease in the spread of source coordinates and lowering of nonuniformities in the distribution of $E_{inc}(x)$. For n = 6 the symmetry requirement directly allowed to obtain a smooth curve of radiator locations. As a result of solving third series problems we succeeded in obtaining, for all n, smooth curves of radiator locations in height over the surface, which agree well with intuitive concepts on optimal solutions. The results of solving problems of the third and fourth series are very close, therefore in Fig. 2 they correspond to the same curves.

As can be expected, a decrease in the number of varying parameters during the imposition of further conditions on radiator locations leads to lowering in the number of steps of optimized search, while in other cases it is quite substantial. The quality of solutions obtained in all cases with account of a priori information is enhanced (δE is decreased). We note that in the case of using a priori information small (up to 5%) variations in $E_{inc}{}^0$ corresponded to equally small (up to 4.5%) changes in y_i obtained.

We note here that account of a priori information, as in any other regularization method, does not provide total guarantee of satisfying the correctness condition. However, the results obtained by numerical modeling make it possible to talk about a high probability of successful use of this approach so as to obtain stable solutions of geometric IPRHT.

Thus, the following conclusions can be drawn on the basis of the studies carried out.

1. In the general case the incorrectness was shown of the geometric inverse problem of heat transfer, related to the determination of optimal radiator arrangement in an RHD from a given field $E_{inc}{}^0$. It was established that the PF relief is a mildly sloping supporting surface with a large number of very small scale oscillations.

2. The use of a priori information in the form of imposing restrictions dictating the arrangements of radiators in an RHD allows to reduce the original incorrect problem to a conditionally correct one, to substantially reduce the number of steps of the optimization search, and correspondingly the total time of computer solution.

The results obtained justify the recommendation of studying regularization methods in the case of using more complicated mathematical models of radiative heat transfer in the practice of numerical parameter optimization of radiative heating devices.

NOTATION

n, number of radiators; N = n − 1, number of zones into which the plate surface is divided; t_0 and t_f, initial and final polyhedron sizes; $\overline{y} = \{y_i\}$ $(i = \overline{1, n})$, vector of vertical coordinates of radiator arrangement; $E_{inc}(x)$, $E_{inc}{}^0(x)$, calculated and assigned flux density distributions of the incident radiation; ℓ, plate thickness; Φ, purpose functional; δE, nonuniformity of flux density distribution of incident radiation; and m_f, final number of steps of the optimization search.

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